Bit Geometrically Uniform Encoders and Applications to Serially Concatenated Trellis Coded Modulation

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Abstract — **A new class of labelings and encoders for which the *Uniform Bit Error Property* holds is introduced, and an application to the design of good serially concatenated TCM schemes is presented.**

serial concatenation

I. Uniform Bit Error Property

A Euclidean-space constellation has the Uniform Error Property (UEP) if the *symbol* error probability does not depend on the transmitted signal. UEP proves very useful for code analysis and design. The Hamming space $\mathbf{H}_k$ is the set of all 2^k binary k-ples. Given a finite constellation $\mathcal{S}$ with cardinality $|\mathcal{S}| = 2^k$, a binary labeling $E[\mathcal{S}, k]$ for $\mathcal{S}$ is a one-to-one function $E : \mathcal{S} \leftrightarrow \mathbf{H}_k$. The bit error probability with Maximum Likelihood (ML) symbol decoding, when a signal $s_i \in \mathcal{S}$ is transmitted, is: $P_b(e|s_T = s_i) = \sum_{j \neq i} \frac{w_H(E(s_j)+E(s_i))}{k} P[s_R = s_j | s_T = s_i]$. A binary labeling $E[\mathcal{S}, k]$ is said to satisfy the Uniform Bit Error Property (UBEP) if the bit error probability with ML symbol decoding does not depend on the transmitted signal, *i.e.*, $P_b(e|s_T = s_i)$ is the same for each signal $s_i \in \mathcal{S}$. A Geometrically Uniform (GU) constellation [1] is a good starting point for UBEP, because the Voronoi regions of the signals are all congruent and the UEP holds. Fixed a signal s_0, a GU constellation usually admits a one-to-one correspondence with a a *generating group* G, so that we will also write the labeling $E[\mathcal{S}, k]$ as $E[G, k]$.

Lemma
Given a GU constellation $\mathcal{S}$ with generating group G, a binary labeling $E[G, k]$ satisfying the following distance rule:

$$d_H(E(g_i), E(g_j)) = w_H(E(-g_i + g_j)) \quad \forall g_i, g_j \in G, \qquad (1)$$

possesse the UBEP.

This labeling will be called a *Bit Geometrically Uniform* (BGU) labeling. There are strong connections between BGU labelings, and Euclidean and Hamming symmetries: it can be proved that $\mathcal{S}$ admits a BGU labeling $E[G, k]$ if and only if G is isomorphic to a generating group of $\mathbf{H}_k$. As an example, an 8-PSK constellation does not admit a generating group isomorphic to $\mathbf{Z}_2^3$ [1], but it admits a BGU Gray labeling based on a generating group of $\mathbf{H}_3$ isomorphic to $\mathbf{D}_4$, the dihedral group of order eight [3]. The BGU property can be easily extended to binary encoders by using a finite-state machine description [3].

II. Serially concatenated TCM

The serial concatenation of an outer binary convolutional encoder with an inner TCM encoder over a multidimensional Euclidean constellation through an interleaver (SCTCM, for brevity), has been introduced in [2]. SCTCM allows to extend the extremely good performance of turbo codes to the case of spectrally efficient coded modulations. In [2] the design approach was based on a "cut-and-try" maximization of the *effective free Euclidean distance* of the inner TCM recursive encoder, defined as the minimum distance between code sequences generated by information sequences that differ only by two bits: $d_{f,\text{eff}} \triangleq \min_{\mathbf{c}_1,\mathbf{c}_2} d_E(\mathbf{c}_1, \mathbf{c}_2)$ for all $\mathbf{c}_1, \mathbf{c}_2 \in C$ with $d_H(E(\mathbf{c}_1), E(\mathbf{c}_2)) = 2$. According to the definition, the computation of $d_{f,\text{eff}}$ requires in general testing of all possible pairs $(\mathbf{c}_1, \mathbf{c}_2)$. However, if the inner TCM encoder is BGU, we can compute $d_{f,\text{eff}}$ as $d_{f,\text{eff}} = \min_{\mathbf{c}} w_E(\mathbf{c})$ for all $\mathbf{c} \in C$ with $w_H(E(\mathbf{c})) = 2$, with a great simplification.

As an example of application of the BGU approach, an SCTCM scheme with spectral efficiency of 2 bps/Hz has been obtained from an outer 2-state, rate 4/5, binary convolutional encoder, a large interleaver, and a 2-state TCM inner encoder defined on a 2×8-PSK constellation with spectral efficiency 2.5 bps/Hz. The code (denoted by A in Fig. 1) obtained after an exhaustive search over the class of BGU encoders has $d_{f,\text{eff}} = 3.76$, which compares very favorably with the heuristic construction of [2] (code B in Fig. 1), leading to $d_{f,\text{eff}} = 1.76$. Analytical upper bounds to the bit error probability, evalutated through an easy (thanks to the BGU property) extension of the techinque based on the *uniform* interleaver (of length 100 and 1000), are reported in Fig. 1 for the two SCTCM schemes.

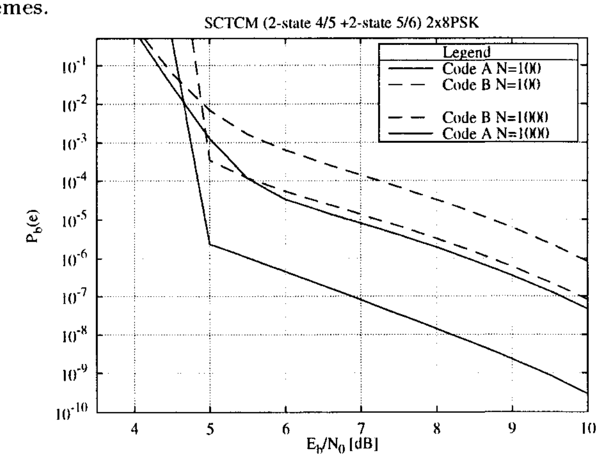

Fig. 1: Upper bounds to the bit error probability for SCTCM codes A and B of spectral efficiency 2 bps/Hz.

References

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